Classical Field Theory of the Photoelectric Effect

Sergey A. Rashkovskiy

1 Introduction

Our current understanding of quantum mechanics is based on certain basic physical effects that, it is believed, cannot be explained within the framework of classical ideas and, therefore, require quantization.

The photoelectric effect has a special place in quantum theory because it became the first physical effect, for explanation of which the quantization of light was introduced.

By the early twentieth century, the three basic laws of the photoelectric effect were experimentally established: (1) the photoelectric current is proportional to the intensity of incident light; (2) the maximum kinetic energy of the emitted photoelectrons varies linearly with the frequency of incident electromagnetic radiation and does not depend on the flux; and (3) for each substance, there is a threshold frequency (the so-called red edge of the photoelectric effect), below which the photoelectric current is not observed.

The second and third laws of the photoelectric effect would appear to contradict classical electrodynamics, which requires dependence of the kinetic energy of the emitted photoelectrons on the intensity of the incident light. Such a conclusion necessarily follows from the analysis of the motion of charged particles—electrons in the field of a classical electromagnetic wave. Thus, the attempts to explain the photoelectric effect within the framework of classical mechanics and classical electrodynamics were unsuccessful.

S. A. Rashkovskiy (✉)
Institute for Problems in Mechanics of the Russian Academy of Sciences, Moscow, Russia

Tomsk State University, Tomsk, Russia
e-mail: rash@ipmnet.ru

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This contradiction was overcome due to the quantization of radiation, which postulates that the absorption of light occurs in the form of discrete quanta $\hbar\omega$ (Einstein 1905). At present, in connection with this finding, it is considered to be generally accepted that the photoelectric effect provides "evidence" for the quantum nature of light.

However, in the early years of quantum mechanics, it was shown that the photoelectric effect is fully described within the framework of so-called semiclassical theory, in which light is considered to be a classical electromagnetic wave, while the atom is quantized and described by the wave equation, e.g. the Schrödinger equation or the Dirac equation [1–4].

In this case, the wave equation is solved as a typical classical field equation, whereby a continuous wave field is calculated. A "quantization" of this wave field occurs only at the stage of interpreting the solution, from which the "probability of photoelectron emission" from an atom is determined.

There were also attempts to build the semiclassical theories of other quantum phenomena, namely, Lamb shift [5, 6], spontaneous emission [5–7], semiclassical radiation theory [8], radiative effects [9], Compton effect [10–15], Hanbury Brown and Twiss effect [16, 17], semiclassical theory of laser [18, 19], etc. Because the electron in such theories is considered to be a quantum particle and light is considered to be a classical electromagnetic field, such theories are considered to be "semiclassical".

Despite the success of this approach, there are many intra-atomic and optical phenomena that did not find an explanation within the framework of semiclassical theory. Because of this, it is generally accepted that a complete description of the intra-atomic phenomena and light-atom interaction is possible only within the framework of quantum electrodynamics (QED), when both the states of an atom and the radiation itself are quantized.

However, as shown in [20–26], there is no need to introduce the quantization of electromagnetic and electron fields because this interpretation is external to the wave equation, and it does not follow from these equations. Moreover, this approach is superfluous in explaining the many physical phenomena that before were interpreted as a result of the quantization of matter.

In previous papers of this series [22–26], an attempt was made to construct a completely classical theory, which is similar to classical field theory [27], in which any quanta are absent. Here, as in [20–26], classical theory is understood as a theory in which all objects are either particles or fields, and no object can simultaneously possess both wave and corpuscular properties. In other words, in classical theory, there is no such concept as corpuscular-wave dualism. Thus, in papers [20–22], it was shown that the discrete events (e.g. clicks of a detector, emergence of the spots on a photographic plate) that are observed in some of the "quantum" experiments with light (especially in the double-slit experiments), which are considered to be direct evidence of the existence of photons, can in fact be explained within classical electrodynamics without quantization of the radiation. Similarly, if the electrons are considered to not be a particle but instead a classical continuous wave field, similar to the classical electromagnetic field, one can consistently explain the "wave-

particle duality of electrons" in the double-slit experiments [23]. In this case, the Dirac equation and its specific cases (Klein-Gordon, Pauli and Schrödinger) should be considered to be the usual field equations of a classical electron wave field, similar to Maxwell's equations for classical electromagnetic fields. As was shown in [23], considering the electron wave as a classical field, we must assign to it, besides the energy and momentum which are distributed in space, also an electric charge, an internal angular momentum and an internal magnetic moment, which are also continuously distributed in space. In this case, the internal angular momentum and internal magnetic moment of the electron wave are its intrinsic properties and cannot be reduced to any movement of charged particles. This viewpoint allows for a description in natural way, in the framework of classical field theory with respect to the many observed phenomena that involve "electrons", and it explains their properties which are considered to be paradoxical from the standpoint of classical mechanics. Thus, the Compton effect, which is considered to be "direct evidence of the existence of photons", has a natural explanation if both light and electron waves are considered to be classical continuous fields [23]. The same approach can be applied to the Born rule for light and "electrons" and to the Heisenberg's uncertainty principle, which have a simple and clear explanation within classical field theory [20–23]. Using such a point of view on the nature of the "electron", a new model of the hydrogen atom that differs from the conventional planetary model was proposed and justified in [24]. According to this model, the atom represents a classical open volume resonator in which an electrically charged continuous electron wave is held in a restricted region of space by the electrostatic field of the nucleus. As shown in [24], the electrostatic field of the nucleus plays for the electron wave, the role of a "dielectric medium", and thus, one can say that the electron wave is held in the hydrogen atom due to the total internal reflection on the inhomogeneities of this "medium". In the hydrogen atom, as in any volume resonator, there are eigenmodes that correspond to a discrete spectrum of eigenfrequencies, which are the eigenvalues of the field equation (e.g. Schrödinger, Dirac). As usual, the standing waves (in this case, the standing electron waves) correspond to the eigenmodes. If only one of the eigenmodes is excited in the atom as in the volume resonator, then such a state of the atom is called a pure state. If simultaneously several (two or more) eigenmodes are excited in the atom, then such a state is called a mixed state [24].

Using this viewpoint, it was shown in [24] that all of the basic optical properties of the hydrogen atom have a simple and clear explanation in the framework of classical electrodynamics without any quantization. In particular, it was shown that the atom can be in a pure state indefinitely. This arrangement means that the atom has a discrete set of stationary states, which correspond to all possible pure states, but only the pure state that corresponds to the lowest eigenfrequency is stable. Precisely this state is the ground state of the atom. The remaining pure states are unstable, although they are the stationary states. Any mixed state of an atom in which several eigenmodes are excited simultaneously is nonstationary, and according to classical electrodynamics, the atom that is in that state continuously emits electromagnetic waves of the discrete spectrum, which is interpreted as a spontaneous emission.

In reference [24], a fully classical description of spontaneous emission was given, and all of its basic properties that are traditionally described within the framework of quantum electrodynamics were obtained. It is shown that the "jump-like quantum transitions between the discrete energy levels of the atom" do not exist, and the spontaneous emission of an atom occurs not in the form of discrete quanta but continuously.

As is well known, the linear wave equation, e.g. the Schrödinger equation, cannot explain the spontaneous emission and the changes that occur in the atom in the process of spontaneous emission (so-called quantum transitions). To explain spontaneous transitions, quantum mechanics, it is believed, must be extended to quantum electrodynamics, which introduces such an object as a QED vacuum, the fluctuations of which are considered to be the cause of the "quantum transitions".

In reference [24], it was shown that the Schrödinger equation, which describes the electron wave as a classical field, is sufficient for a description of the spontaneous emission of a hydrogen atom. However, it should be complemented by a term that accounts for the inverse action of self-electromagnetic radiation on the electron wave. In the framework of classical electrodynamics, it was shown that the electron wave as a classical field is described in the hydrogen atom by a nonlinear Eq. [24]

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m_e}\Delta\psi - \frac{e^2}{r}\psi - \frac{2e^2}{3c^3}\psi\mathbf{r}\frac{\partial^3}{\partial t^3}\int \mathbf{r}|\psi|^2 d\mathbf{r} \qquad (1)$$

where the last term on the right-hand side describes the inverse action of the self-electromagnetic radiation on the electron wave and is responsible for the degeneration of any mixed state of the hydrogen atom. Precisely, this term "provides" a degeneration of the mixed state of the hydrogen atom to a pure state, which corresponds to the lower excited eigenmodes of an atom. As shown in [24], this term has a fully classical meaning and fits into the concept developed in [20–26] in that the photons and electrons as particles do not exist, and there are only electromagnetic and electron waves, which are classical (continuous) fields.

The nonlinearity of the Eq. (1) plays an essential role in light-atom interaction and should be taken into account in all calculations. Thus, as shown in [25, 26], based on the nonlinear Eq. (1), the light-atom interaction can be fully described within the framework of classical field theory without the use of quantum electrodynamics. In particular, in reference [25], the optical Bloch equations with damping due to spontaneous emission and with correct damping rate has been directly derived from the nonlinear Schrödinger equation (1) without quantization of radiation [25].

In reference [26] it was shown that the thermal radiation can also be described without quantization of energy in the framework of classical field theory using the nonlinear Schrödinger equation (1) which is considered as a classical field equation. As shown in [26] the Planck's law for the spectral energy density of thermal radiation and the Einstein A-coefficient for spontaneous emission are derived without using the concept of the energy quanta.

As will be shown below, the failures of classical electrodynamics in explaining the photoelectric effect are connected with the incorrect postulate that electrons

are particles. I will show that for a consistent explanation of the photoelectric effect within the framework of classical field theory, it is sufficient to abandon this postulate and consider continuous classical electron waves instead of the particles-electrons [23, 24]. The considered theory is fully classical because it does not contain not only the quantization of the radiation but also the quantization of the electron wave.

2 Photoelectric Effect

In reference [25], it was assumed that under the influence of an incident electromagnetic wave, the electron wave in an atom is only redistributed between its eigenmodes but not emitted outward by the atom. In this case, internal electric currents arise inside the atom that, however, cannot be detected by macroscopic devices. Such a situation occurs at a relatively low frequency of the incident electromagnetic wave. If this frequency is sufficiently large, then an emission of the electron wave by the atom occurs. Because the electron wave has an electric charge that is continuously distributed in space [23, 24], in this case, an external electric current (photoelectric current) appears that can be detected by macroscopic devices. As a result, the photoelectric effect will be observed.

From the considered point of view [20–26], the photoelectric effect represents an emission of the continuous charged electron wave by an atom that was excited by the incident classical electromagnetic wave. Formally, the photoelectric effect is no different from the stimulated emission of electromagnetic waves by an atom [24], with the only difference being that the electron wave emitted by an atom is electrically charged, while the electromagnetic wave does not carry the electric charge. Assuming that the electric charge is continuously distributed in the electron wave [23, 24], one concludes that in the process of the emission of the electron wave, the atom is positively charged continuously. However, accounting for the fact that the electron wave for an as yet inexplicable reason does not "feel" its own electrostatic field [24], this process will not affect the emission of the following "portions" of the continuous electron wave because they must overcome the same electrostatic potential of the nucleus.

Let us consider the photoelectric effect for the hydrogen atom being in the classic monochromatic electromagnetic wave.

In this section, we neglect the inverse action on the electron wave of its own nonstationary electromagnetic field. For this reason, the last term in the Schrödinger equation (1), which is associated with a spontaneous emission of the electromagnetic waves, will not be considered, and we will use the conventional linear Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m_e}\Delta\psi - \frac{e^2}{r}\psi + \psi e\mathbf{r}\mathbf{E}_0\cos\omega_0 t \tag{2}$$

where ω_0 is the frequency of the incident light. We will consider here the approximation, when the wavelength of the incident electromagnetic wave is substantially larger than the characteristic spatial size of the electron field in the hydrogen atom, which is of the order of the Bohr radius a_B.

The wave function of an electron wave can be represented as in [28]

$$\psi = \sum_k c_k(t) u_k(\mathbf{r}) \exp(-i\omega_k t) + \sum_n \int_0^\infty C_n(\omega, t)\, f_n(\mathbf{r}, \omega) \exp(-i\omega t)\, d\omega \quad (3)$$

where the first sum describes that part of the electron wave that is contained in the eigenmodes of the atom (i.e. corresponding to a "finite motion" of the electron wave), and for this term, all $\omega_k < 0$, while the integrals describe the electron waves that are emitted by an atom (i.e. which corresponds to the "infinite motion" of the electron wave), to which it is known that $\omega > 0$ corresponds. The indices n and k run through the appropriate integer values. The functions $u_k(\mathbf{r})$ and $f_n(\mathbf{r}, \omega)$ are the eigenfunctions of the stationary Schrödinger equation, while the frequencies ω_k are the eigenvalues that correspond to the eigenfunctions $u_k(\mathbf{r})$.

The eigenfunctions $u_k(\mathbf{r})$ and $f_n(\mathbf{r}, \omega)$ satisfy the orthogonality conditions:

$$\int u_k(\mathbf{r})\, u_n^*(\mathbf{r})\, dV = \delta_{nk} \quad (4)$$

$$\int f_k(\mathbf{r}, \omega')\, f_n^*(\mathbf{r}, \omega'')\, dV = \delta_{nk}\delta(\omega' - \omega'') \quad (5)$$

$$\int u_k(\mathbf{r})\, f_n^*(\mathbf{r}, \omega'')\, dV = 0 \quad (6)$$

Substituting expression (3) into Eq. (2) and using the orthogonality conditions (4)–(6), we obtain

$$i\hbar \dot{c}_k(t) \exp(-i\omega_k t) = e\mathbf{E}_0 \cos\omega_0 t \sum_n c_n(t) \int \mathbf{r} u_n(\mathbf{r})\, u_k^* dV \exp(-i\omega_n t)$$
$$+ e\mathbf{E}_0 \cos\omega_0 t \sum_n \int_0^\infty C_n(\omega, t) \int \mathbf{r} u_k^* f_n(\mathbf{r}, \omega)\, dV \exp(-i\omega t)\, d\omega \quad (7)$$

and

$$i\hbar \dot{C}_n(\omega, t) \exp(-i\omega t) = e\mathbf{E}_0 \cos\omega_0 t \sum_k c_k(t) \int \mathbf{r} f_n^*(\mathbf{r}, \omega)\, u_k(\mathbf{r})\, dV \exp(-i\omega_k t)$$
$$+ e\mathbf{E}_0 \cos\omega_0 t \sum_k \int_0^\infty C_k(\omega', t) \int \mathbf{r} f_k(\mathbf{r}, \omega')\, f_n^*(\mathbf{r}, \omega)\, dV \exp(-i\omega' t)\, d\omega' \quad (8)$$

Within the framework of perturbation theory and assuming that all of the modes of the electron wave (both discrete and continuous), except for the ground mode u_1, are weakly excited, we obtain

$$i\hbar\dot{c}_1(t)\exp(-i\omega_1 t) = -c_1(t)\,(\mathbf{E}_0\mathbf{d}_{11})\cos\omega_0 t\exp(-i\omega_1 t) + e\mathbf{E}_0\cos\omega_0 t\sum_n\int_0^\infty C_n(\omega,t)\int\mathbf{r}u_1^* f_n(\mathbf{r},\omega)\,dV\exp(-i\omega t)\,d\omega \tag{9}$$

$$i\hbar\dot{C}_n(\omega,t) = e\mathbf{E}_0\cos\omega_0 t c_1(t)\exp[-i(\omega_1-\omega)t]\int\mathbf{r}f_n^*(\mathbf{r},\omega)u_1(\mathbf{r})\,dV + e\mathbf{E}_0\cos\omega_0 t\exp(i\omega t)\sum_k\int_0^\infty C_k(\omega',t)\int\mathbf{r}f_k(\mathbf{r},\omega')f_n^*(\mathbf{r},\omega)\,dV\exp(-i\omega' t)\,d\omega' \tag{10}$$

where

$$\mathbf{d}_{nk} = -e\int\mathbf{r}u_n(\mathbf{r})u_k^* dV \tag{11}$$

For a weak electromagnetic wave, which causes weak excitation of an atom, $c_1 \approx 1$. For this reason, we can discard terms in Eq. (10) that contain $\mathbf{E}_0 C_k(\omega', t)$, as small of the second order. In Eq. (9), these terms cannot be discarded because the change in c_1 will have a second order in $\mathbf{E}_0$. Then, we obtain

$$i\hbar\dot{c}_1(t) = -(\mathbf{E}_0\mathbf{d}_{11})\cos\omega_0 t + \frac{1}{2}e\mathbf{E}_0\sum_n\int_0^\infty C_n(\omega,t)\mathbf{U}_{1n}(\omega)\exp[-i(\omega-\omega_1-\omega_0)t]\,d\omega + \frac{1}{2}e\mathbf{E}_0\sum_n\int_0^\infty C_n(\omega,t)\mathbf{U}_{1n}(\omega)\exp[-i(\omega-\omega_1+\omega_0)t]\,d\omega \tag{12}$$

$$i\hbar\dot{C}_n(\omega,t) = \frac{1}{2}e\mathbf{E}_0\cdot\mathbf{U}_{1n}^*(\omega)\exp[-i(\omega_1-\omega-\omega_0)t] + \frac{1}{2}e\mathbf{E}_0\cdot\mathbf{U}_{1n}^*(\omega)\exp[-i(\omega_1-\omega+\omega_0)t] \tag{13}$$

where

$$\mathbf{U}_{1n}(\omega) = \int\mathbf{r}u_1^*(\mathbf{r})\,f_n(\mathbf{r},\omega)\,dV \tag{14}$$

Neglecting the purely oscillatory term $-(\mathbf{E}_0\mathbf{d}_{11})\cos\omega_0 t$ in Eq. (12) (which can be accomplished, for example, by averaging Eq. (12) over rapid oscillations with a frequency ω_0), we obtain

$$i\hbar\dot{c}_1(t) = \frac{1}{2}e\mathbf{E}_0\sum_n\int_0^\infty C_n(\omega,t)\,\mathbf{U}_{1n}(\omega)\exp\left[-i(\omega-\omega_1-\omega_0)t\right]d\omega$$

$$+\frac{1}{2}e\mathbf{E}_0\sum_n\int_0^\infty C_n(\omega,t)\,\mathbf{U}_{1n}(\omega)\exp\left[-i(\omega-\omega_1+\omega_0)t\right]d\omega \tag{15}$$

Integrating Eq. (13) with respect to the time from zero to t, we obtain

$$C_n(\omega,t) = \frac{e}{2\hbar}\frac{\exp\left[-i(\omega_1-\omega-\omega_0)t\right]-1}{(\omega_1-\omega-\omega_0)}\mathbf{E}_0\cdot\mathbf{U}_{1n}^*(\omega)$$

$$+\frac{e}{2\hbar}\frac{\exp\left[-i(\omega_1-\omega+\omega_0)t\right]-1}{(\omega_1-\omega+\omega_0)}\mathbf{E}_0\cdot\mathbf{U}_{1n}^*(\omega) \tag{16}$$

Because the frequencies have $\omega_0 > 0$, $\omega > 0$ and $\omega_1 < 0$, the value $\omega_1 - \omega - \omega_0$ is not equal to zero for any ω, and thus, the first term will always be limited and will describe the oscillations that are of small amplitude. At the same time, $\omega_1 - \omega + \omega_0 = 0$ at the resonance frequency of $\omega_0 = |\omega_1| + \omega$, and near the resonant frequency, the second term in (16) will increase indefinitely. Therefore, the second term in (16) makes the main contribution to the effect that is under consideration. Neglecting the first term in expression (16), we obtain

$$C_n(\omega,t) = \frac{e}{2\hbar}\frac{\exp\left[-i(\omega_1-\omega+\omega_0)t\right]-1}{(\omega_1-\omega+\omega_0)}\mathbf{E}_0\cdot\mathbf{U}_{1n}^*(\omega) \tag{17}$$

Let us calculate the photoelectric current that arises upon excitation of the atom by the incident electromagnetic wave.

This goal can be accomplished by calculating the electric current density according to the formula

$$\mathbf{j} = i\frac{ec^2}{2\omega_e}\left(\psi^*\nabla\psi - \psi\nabla\psi^*\right) - \frac{e^2c}{\hbar\omega_e}\mathbf{A}\psi\psi^* \tag{18}$$

and integrating it over the surface of an infinite sphere whose centre is in the nucleus of the atom. However, it is more convenient to accomplish this step while using the law of conservation of charge and accounting for the fact that $q_k = -e|c_k|^2$ is the electric charge that is contained in mode k of the electron wave [24]. Then, $\dot{q}_k$ is the internal electric current in the atom, by which mode k is exchanged with all of the other modes of the electron wave (including continuous modes, if they exist), i.e.

the amount of electric charge of the electron wave, which goes into mode k from other modes or goes out of mode k into other modes, per unit time. Because in this case, it is considered that only one (ground) eigenmode u_1 of the hydrogen atom is excited, then the photoelectric current

$$I_{ph} = -\dot{q}_1 \tag{19}$$

or

$$I_{ph} = -e\frac{d|c_1|^2}{dt} \tag{20}$$

Using Eq. (15), we obtain the same approximation

$$\begin{aligned} I_{ph} = &-\frac{e^2}{2i\hbar}\sum_n \int_0^\infty C_n(\omega,t)\,\mathbf{E}_0\cdot\mathbf{U}_{1n}(\omega)\exp\left[-i(\omega-\omega_1-\omega_0)t\right]d\omega \\ &-\frac{e^2}{2i\hbar}\sum_n \int_0^\infty C_n(\omega,t)\,\mathbf{E}_0\cdot\mathbf{U}_{1n}(\omega)\exp\left[-i(\omega-\omega_1+\omega_0)t\right]d\omega \\ &+\frac{e^2}{2i\hbar}\sum_n \int_0^\infty C_n^*(\omega,t)\,\mathbf{E}_0\cdot\mathbf{U}_{1n}^*(\omega)\exp\left[i(\omega-\omega_1-\omega_0)t\right]d\omega \\ &+\frac{e^2}{2i\hbar}\sum_n \int_0^\infty C_n^*(\omega,t)\,\mathbf{E}_0\cdot\mathbf{U}_{1n}^*(\omega)\exp\left[i(\omega-\omega_1+\omega_0)t\right]d\omega \end{aligned} \tag{21}$$

Substituting $C_n(\omega,t)$ from (17) into expression (21), we obtain

$$\begin{aligned} I_{ph} = &\frac{e^3}{2\hbar^3}\sum_n \int_0^\infty \frac{\sin\left[(\omega-\omega_1-\omega_0)t\right]}{(\omega-\omega_1-\omega_0)}\left(\mathbf{E}_0\cdot\mathbf{U}_{1n}^*\right)(\mathbf{E}_0\cdot\mathbf{U}_{1n})\,d\omega \\ &+\frac{e^3}{4i\hbar^3}\exp(-2i\omega_0 t)\sum_n \int_0^\infty \frac{1-\exp\left[-i(\omega-\omega_1-\omega_0)t\right]}{(\omega-\omega_1-\omega_0)}\left(\mathbf{E}_0\cdot\mathbf{U}_{1n}^*\right)(\mathbf{E}_0\cdot\mathbf{U}_{1n})\,d\omega \\ &-\frac{e^3}{4i\hbar^3}\exp(2i\omega_0 t)\sum_n \int_0^\infty \frac{1-\exp\left[i(\omega-\omega_1-\omega_0)t\right]}{(\omega-\omega_1-\omega_0)}(\mathbf{E}_0\cdot\mathbf{U}_{1n})\left(\mathbf{E}_0\cdot\mathbf{U}_{1n}^*\right)d\omega \end{aligned} \tag{22}$$

The second and third terms on the right-hand side of expression (22) are rapidly oscillating at a frequency of ω_0, and they can be discarded by averaging over the fast oscillations. Then, we obtain

$$I_{ph} = \frac{e^3}{2\hbar^2}\int\limits_0^\infty \frac{\sin\left[(\omega-\omega_1-\omega_0)\,t\right]}{(\omega-\omega_1-\omega_0)}\sum_n \left(\mathbf{E}_0\cdot\mathbf{U}_{1n}^*\right)\left(\mathbf{E}_0\cdot\mathbf{U}_{1n}\right)d\omega \tag{23}$$

Assuming that all of the orientations of the atom in space are equally probable and therefore the vector $\mathbf{U}_{1n}$ is statistically isotropic, one averages the current (23) overall possible orientations of the atom.

Then,

$$\overline{\left(\mathbf{E}_0\mathbf{U}_{1n}\right)\left(\mathbf{E}_0\mathbf{U}_{1n}^*\right)} = E_{0i}E_{0j}\overline{U_{1n,i}U_{1n,j}^*} \tag{24}$$

where the bar denotes averaging over all possible orientations and the indices i and j are the vector indexes.

For the isotropic vector $\mathbf{U}_{1n}$,

$$\overline{U_{1n,i}U_{1n,j}^*} = \frac{1}{3}|\mathbf{U}_{1n}|^2\delta_{ij} \tag{25}$$

Then,

$$\overline{\left(\mathbf{E}_0\mathbf{U}_{1n}\right)\left(\mathbf{E}_0\mathbf{U}_{1n}^*\right)} = \frac{1}{3}|\mathbf{E}_0|^2|\mathbf{U}_{1n}|^2 \tag{26}$$

Accordingly, for the mean photoelectric current (23), we obtain

$$\overline{I_{ph}} = \beta|\mathbf{E}_0|^2 \tag{27}$$

where the parameter

$$\beta = \frac{e^3}{6\hbar^2}\int\limits_0^\infty \frac{\sin\left[(\omega-\omega_1-\omega_0)\,t\right]}{(\omega-\omega_1-\omega_0)}\sum_n |\mathbf{U}_{1n}(\omega)|^2 d\omega \tag{28}$$

does not depend on the incident light intensity $|\mathbf{E}_0|^2$ and instead, the parameter β depends on the frequency ω_0 of the incident light.

Thus, we have obtained the first law of the photoelectric effect without using the photon hypothesis within the framework of only classical field theory while considering the electromagnetic and electron waves as classical fields.

Let us consider the dependence of the parameter β on the frequency of the incident light ω_0.

Let us denote

$$F(\omega) = \sum_n |\mathbf{U}_{1n}(\omega)|^2 \tag{29}$$

$$x = \omega - \omega_1 - \omega_0 \tag{30}$$

Then, we obtain

$$\beta = \frac{e^3}{6\hbar^2} \int\limits_{|\omega_1|-\omega_0}^{\infty} \frac{\sin(xt)}{x} F(x - |\omega_1| + \omega_0)\, dx \tag{31}$$

Here, we account for the fact that $\omega_1 < 0$.

The function $\frac{\sin(xt)}{x}$ has a sharp peak in the vicinity of $x = 0$ and has a width of $\Delta x \sim \pi/t$, and at $t \to \infty$, it behaves similar to a delta-function: $\int_{-\infty}^{\infty} \frac{\sin(xt)}{x} dx = \pi$. The function $F(\omega)$ in the vicinity of $x = 0$ is smooth and varies weakly on the interval $\Delta x \sim \pi/t$.

Therefore, with reasonable accuracy at $\omega_0 < |\omega_1| - \frac{\pi}{2t}$, we can write

$$\beta \approx \frac{e^3}{6\hbar^2} F(0) \int\limits_{|\omega_1|-\omega_0}^{\infty} \frac{\sin(xt)}{x} dx \tag{32}$$

At the same time, at $\omega_0 - |\omega_1| \gg \frac{\pi}{2t}$, it is necessary to account for the fact that a small neighbourhood of the point $x = 0$ will make the main contribution to the integral in (31) (due to the delta-like behaviour of the integrand). As a result, for $\omega_0 - |\omega_1| \gg \frac{\pi}{2t}$, we obtain

$$\beta \approx \frac{\pi e^3}{6\hbar^2} F(\omega_0 - |\omega_1|) \tag{33}$$

In this case, the parameter β will vary with the frequency of the incident light ω_0.

Figure 1 shows, in a nondimensional form, the dependence of the parameter β on the frequency difference $\omega_0 - |\omega_1|$ in the vicinity of the frequency $\omega_0 = |\omega_1|$.

We can see that the parameter β is virtually zero at $\omega_0 < |\omega_1| - \frac{\pi}{2t}$, and it almost linearly varies from zero to $\frac{\pi e^3}{6\hbar^2} F(0)$ when ω_0 changes in the range from $|\omega_1| - \frac{\pi}{2t}$ to $|\omega_1| + \frac{\pi}{2t}$, and it virtually equals the value in (33) at $\omega_0 > |\omega_1| + \frac{\pi}{2t}$. The width of the frequency range in which there is a noticeable change in the parameter β is $\Delta\omega_0 \sim \pi/t$.

Assuming $|\omega_1| \sim 10^{14}$ rad/s (which corresponds to visible light) for the observation time $t > 10^{-9}$ s, we obtain $\Delta\omega_0 < 3 \cdot 10^9$ rad/s, which is significantly less than $|\omega_1|$:

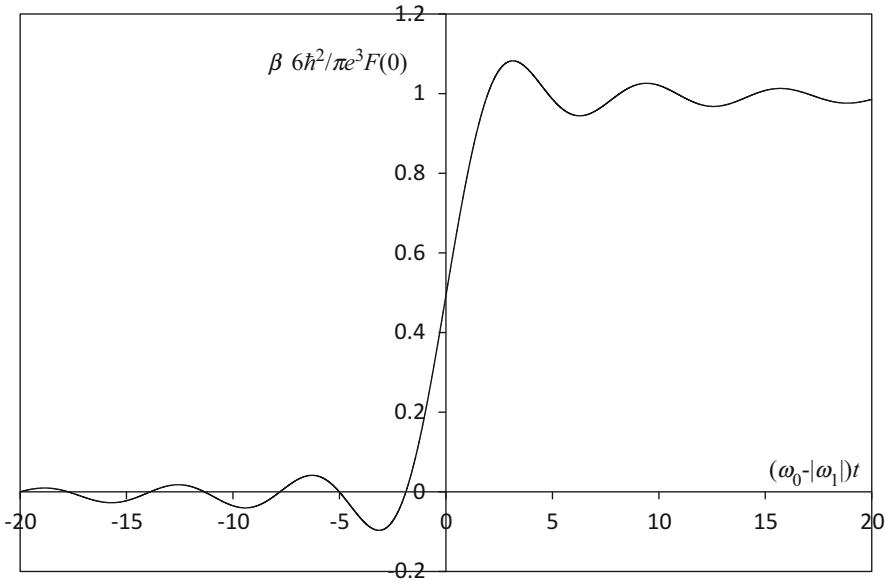

Fig. 1 The dependence of the parameter β on the frequency difference $\omega_0 - |\omega_1|$ in the vicinity of the frequency $\omega_0 = |\omega_1|$

$$\Delta\omega_0 \ll |\omega_1| \tag{34}$$

From this analysis, it follows that for the actual duration of the observation, the parameter β will have almost a threshold dependence on the frequency of the incident light ω_0: for $\omega_0 < |\omega_1|$, we obtain $\beta \approx 0$, and the photoelectric current is almost absent, while at $\omega_0 > |\omega_1|$, the parameter β will take the value in (33), and the photoelectric current (27) will be proportional to the intensity of the incident light.

Thus, we have obtained the third law of the photoelectric effect also without using the photon hypothesis, within only the framework of classical field theory.

Let us now consider the second law of the photoelectric effect. In its conventional form, it establishes the dependence of the kinetic energy of the emitted photoelectrons on the frequency and intensity of the incident radiation. However, in the experiments on the photoelectric effect, the kinetic energy of the photoelectron is not measured directly; it is determined indirectly through the measured stopping potential. Therefore, such wording of the second law of the photoelectric effect already contains some interpretation of the experimental facts; in particular, it assumes that the electrons are indivisible particles that, at the time of escape from the atom, have a definite kinetic energy. In this case, the kinetic energy of the photoelectrons can be determined through the stopping potential at which the photoelectric current is terminated.

Because in the papers of this series we doubt that electrons are particles, it does not make sense to talk about the kinetic energy of the electrons, and we will need a different formulation of the second law of the photoelectric effect.

To rule out any interpretation of the experimental data, the wording of the second law of the photoelectric effect (and in general, of any laws) should use only measured parameters. From this perspective, an *objective formulation of the second law of the photoelectric effect* will be as follows: the stopping potential varies linearly with the frequency of the incident electromagnetic radiation and does not depend on the flux.

Let us consider the function in (17). The square of its modulus $|C_n(\omega, t)|^2$ determines the density of the photoelectric current (18). This function reaches its maximum when

$$\omega_1 - \omega + \omega_0 = 0 \tag{35}$$

and for large t, the largest part of the photoelectric current falls on the narrow range of the frequencies of the electron wave that have the width

$$\Delta\omega \sim \pi / t \tag{36}$$

near the frequency

$$\omega = \omega_0 - |\omega_1| \tag{37}$$

When accounting for the smallness of the frequency range (36), it can be assumed that the electron wave that is emitted by an atom is almost monochromatic and has the frequency in (37), which linearly depends on the frequency of the incident light ω_0 and does not depend on its intensity.

Let us place on the path of the electron wave a decelerating potential. In this case, we come to the problem of propagation of the electron wave in the field of the decelerating potential, which is quite accurately described by the linear Schrödinger equation. At large distances from the atom, the electron wave can be considered to be approximately flat. To simplify the analysis, instead of the decelerating potential, having a linear dependence on the coordinates along which the electron wave propagates, let us consider the potential step (barrier) of the same "height" U_0 and the same width L to be the actual decelerating potential. The solution of the Schrödinger equation for the potential step is well known [28]: at $\hbar\omega > U_0$, the electron wave passes through a potential step and is partially reflected from it, while when $\hbar\omega < U_0$, the electron wave is mainly reflected from the potential step, although a small part goes through the potential step due to tunnelling. The transmission coefficient of the electron wave for the potential step (in our interpretation, this coefficient is the ratio of the electric current of the electron wave behind the potential step to the electric current of the electron wave arriving to the potential steps from an atom) in the limiting case $\hbar\omega = U_0$ is defined by the expression [28]

$$D = \left(1 + \frac{2m_e \omega L^2}{4\hbar}\right)^{-1} \tag{38}$$

Here, instead of the energy of a non-relativistic quantum particle, we use a Schrödinger frequency ω (which is equal to the difference between the true frequency of the electron wave that is entered into the solution of the Dirac equation and its "rest frequency" $\omega_e = mc^2/\hbar$ [23]). With the increase in the width of the potential step L, the transmission coefficient (38) decreases rapidly, and for an actual decelerating potential that has macroscopic sizes that substantially exceed the de Broglie wavelength $\lambda_{dB} = 2\pi\sqrt{\frac{\hbar}{2m_e\omega}}$, it is almost equal to zero because, in this case, we can neglect the tunnelling.

Thus, for the macroscopic decelerating potentials that are used in the experiments, there is a threshold effect: when $\hbar\omega > U_0$, the electron wave passes through the decelerating potential, while when $\hbar\omega \leq U_0$, the electron wave is fully "reflected" by the decelerating potential and the photoelectric current is not observed behind it. This arrangement means that there is a limit to the value of the decelerating potential, which is the stopping potential

$$U_s = \hbar\omega \tag{39}$$

above which the photoelectric current is absent.

When accounting for expression (37), we obtain

$$U_s = \hbar\omega_0 - \hbar\,|\omega_1| \tag{40}$$

This result completely coincides with the above given formulation of the second law of the photoelectric effect, and it was obtained within the framework of classical field theory without the use of such concepts as photons and electrons.

Note that expression (40) can be formally written in the form

$$\hbar\omega_0 = E + A \tag{41}$$

where the notations $A = \hbar|\omega_1|$ and $E = U_s$ were introduced. The expression in (41) can be considered to be Einstein's equation for the photoelectric effect, and one can interpret it within the framework of the photon-electron representations in which the parameter E is interpreted as the kinetic energy of the photoelectrons, while the parameter A is interpreted as a work function of the atom. However, this approach is no more than an interpretation that is based on the formal similarity of the pure wave expression (40) and the mechanical law of energy conservation.

The above analysis has shown that such a corpuscular interpretation of the photoelectric effect is superfluous.

The well-known experiments by Meyer and Gerlach on the photoelectric effect on the particles of metal dust, irradiated with ultraviolet light, are considered to

be one of the pieces of "irrefutable evidence" that light energy is propagated in the form of identical indivisible quanta (photons). Assuming that the electrons are particles while light is composed of continuous classical electromagnetic waves, we can calculate the time during which the metal particle will absorb a sufficient amount of energy for the ejection of an electron. In the experiments by E. Meyer and W. Gerlach, this duration was of the order of a few seconds, which means that the photoelectron cannot leave a speck of dust earlier than in a few seconds after the start of irradiation. In contrast to this conclusion, the photoelectric current in these experiments began immediately after the beginning of the irradiation. Hence, it is usually concluded that this finding is only possible if the light is a flux of photons each of which can be absorbed by the atom only entirely and, therefore, can "knock out" the electron from the atoms at the moment of its collision with the metal particle.

However, this conclusion follows only in the case in which the electrons are considered to be indivisible particles. If instead of considering the electrons to be particles we consider a continuous electron wave [23, 24], then as was shown above, the photoelectric current appears almost without delay after the start of irradiation of an atom by the classical electromagnetic wave and occurs even at very low light intensities, when the light frequency exceeds the threshold frequency for the given atom. This finding is because to start the photoelectric current, the atom does not need to accumulate the energy that is equal to the ionization potential because the electron wave is emitted by the atom continuously and not in the form of discrete portions—"electrons". Note that precisely the need to explain the ejection of discrete electrons from an atom under the action of light led A. Einstein to the idea of light quanta, which when absorbed, gave to the atom sufficient energy for the liberation of a whole electron.

The above analysis shows that all three laws of the photoelectric effect only approximately reflect its actual regularities. In particular, the photoelectric current appears and disappears non-abruptly when "passing" through the threshold frequency $|\omega_1|$, and it gradually increases or decreases in the frequency range that has the width $\Delta\omega_0 \sim \pi/t$ near the threshold frequency $|\omega_1|$. However, this effect can be detected only for ultrashort observation times of $t \sim 10^{-15}$ s, which is difficult to achieve in the experiments on the photoelectric effect. Moreover, consideration of the nonlinear effects in the interaction of the light wave with an atom shows [29] that the photoelectric current appears even in the case when the frequency of the incident light is significantly less than the threshold frequency $|\omega_1|$, which is predicted by the linear theory. Such effects can be observed only in a very intense laser field [30]. Strictly speaking, the theory [29], which describes the ionization of an atom in an intense laser field, is fully classical in the sense under consideration because an atom is described by the Schrödinger equation, while the light wave is considered to be a classical electromagnetic field. The true result of this finding is the photoelectric current that is created by the continuous electron wave emitted by an atom because precisely the photoelectric current is calculated in the theory [29]. However, traditionally, the results of the theory [29] are interpreted from the standpoint of photon-electron representations, which make it necessary

to interpret the main result of the theory [29] as the probability of the ionization of an atom (i.e. the probability of the liberation of an "electron" from the atom) per unit time. The representations with respect to the multiphoton ionization of an atom, when the atom "absorbs simultaneously several photons", the total energy of which exceeds the ionization potential of the atom, were a consequence of such an interpretation. When there is a requirement for too many "photons" for the liberation of the "electron", talking about the simultaneous absorption of such a large number of particles becomes meaningless (because of the low probability of this process); then, the results of the theory [29] are interpreted as a tunnel ionization in which the intense laser field changes the potential field in which the "electron" is positioned, which gives it the "opportunity" to leave the atom due to tunnelling. From the point of view of the ideas that are developed in this series of papers, both "multiphoton" and "tunnel" ionization of an atom are the result of the same process—the interaction of a classical electromagnetic wave with a classical electron wave.

Finally, note that there is no difficulty in calculating the angular distribution of the photoelectric current in the framework of the theory under consideration, if we account for the fact that the continuous electric current created by the electron wave emitted by an atom under the action of light is calculated by expression (18) using the wave function in (3) and (17). Once again, note that this current is not the distribution over the directions of the particles-electrons that are emitted by an atom but the distribution over the directions of the current of a continuous charged electron wave that is emitted by the atom. All of the known expressions that are obtained earlier for the photoelectric effect (see, e.g. [4, 31]) remain valid, but they should now be interpreted from the standpoint of classical field theory.

3 Concluding Remarks

Thus, we see that the light-atom interaction including the photoelectric effect is fully described within the framework of classical field theory without the use of quantum electrodynamics and, in general, without any quantization. The results of this theory utilize the simple classical sense and do not require the postulation of such paradoxical properties of matter as the wave-particle duality. The paradoxes in the theory appear when a continuous light beam or a continuous charged electron wave emitted by the atoms under the influence of incident light is attempted to be interpreted as the flux of indivisible particles—photons or electrons. In this case, the probabilistic interpretation of the results of the theory arises from a need. However, as was shown in this paper and in the previous papers of this series [23–26], the processes that are under consideration are fully deterministic, while the postulate about the probabilistic nature of all quantum phenomena is the result of misinterpretation.

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